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Financial Innovation and Asset Price Volatility†

By Felix Kubler and Karl Schmedders*

A popular sentiment in the aftermath of the financial crisis of 2007–2009 has been that financial innovations do not always lead to better consumption smoothing but instead may give rise to speculative trading and excess asset price volatility. This view is by no means new and had also emerged after previous financial crises. For example, after the stock market crash of 1987 the Report of the Presidential Task Force on Market Mechanisms (1988), better known as the "Brady Report," laid significant blame for the crash on new financial products implementing portfolio insurance mechanisms. While the empirical evidence on the impact of derivatives on the price volatility of the underlying asset is mixed (see, for example, Rahman 2001), the sentiment that derivatives encourage speculation which destabilizes the underlying spot market remains popular.

In this article we present a canonical and parsimonious overlapping generations (OLG) model that supports the popular view. In a version of Huffman's (1987) stochastic OLG model, we compare asset prices for incomplete and complete markets. Individuals within a generational cohort have heterogeneous beliefs about future states of the economy and, thus, would like to make "bets" against each other. In the incomplete-markets economy, the agents cannot make such bets. Their beliefs do not affect the equilibrium, and the resulting asset price volatility is very small. The situation changes dramatically when markets are completed through financial innovations. Now the set of securities is sufficiently rich for the agents with different beliefs to place large bets against each other and, as a result, wealth shifts across agents and across generations. Such changes in the wealth distribution strongly affect asset prices, since older generations have a much higher propensity to consume than younger generations and, as a result, have much stronger incentives to divest of their asset investments. Put differently, prices of long-lived securities are considerably lower when "old" generational cohorts hold most of the wealth than when "young" cohorts hold most of the wealth in the economy. In sum, belief heterogeneity leads to considerable changes in the wealth distribution which, in turn, result in substantial asset price volatility in the complete-markets economy.

I. An OLG Model

We consider an overlapping generations exchange economy with discrete time periods, $t = 0, 1, \ldots$. In each time period two agents enter the economy; they trade and consume for three periods and then leave the economy. An individual is identified by the date of his birth, t, and his type, $h = 1, 2$. We call individuals in their first, second, and third period in the economy young, middle-aged, and old, respectively. In each time period t an exogenous shock $s_t \in \mathcal{S} = \{1, 2\}$ realizes. The individuals entering the economy in period t have an initial endowment $e_t^h = e^h(s_t) > 0$ of the consumption good and receive no further endowments in the subsequent periods. The aggregate endowment of the young agents is denoted by $e_t = e(s_t) = e^1(s_t) + e^2(s_t)$. An agent of type h entering the economy in period t has an intertemporal time-separable expected utility function,

$$U^{t,h}(c) = \ln(c_t^{y,h}) + E_t[\ln(c_{t+1}^{m,h}) + \ln(c_{t+2}^{o,h})],$$

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where $c_t^{y,h}$, $c_{t+1}^{m,h}$, and $c_{t+2}^{o,h}$ denote the agent's consumption in period t when he is young (y), in period $t+1$ when he is middle-aged (m), and in period $t+2$ when he is old (o). The term E_t denotes the expectation conditional on the available information at time t when the individual enters the economy. Agents of type h believe that the exogenous shocks are i.i.d. with subjective probabilities $\pi_{s_t}^h > 0$ for $s_t \in \{1,2\}$.

We examine this OLG economy for two different specifications of financial markets. In the first specification, markets are incomplete; in the second markets are complete. In both economies, agents can trade a stock ("Lucas tree") in order to smooth consumption. The stock is in unit net supply paying dividends $d_t = d(s_t) > 0$ depending on the exogenous shock s_t. We denote the price of the stock by p_t and the end-of-period holdings of young (middle-aged) individuals by $\phi_t^{y,h}(\phi_t^{m,h})$. In the incomplete-markets economy, there are no assets besides the stock.

Since there are only two possible shocks in the economy in each period, markets will generally be complete if another security such as a risk-free bond or an option on the stock is available for trade. To simplify the exposition of the main idea and to avoid additional notation, we consider even simpler securities in the complete-markets economy. In addition to the stock, two Arrow securities are tradeable on financial markets; the first (second) Arrow security pays one unit of the consumption good in the subsequent period if the first (second) shock occurs and nothing otherwise. We denote the prices of the two Arrow securities by $q_{1,t}$ and $q_{2,t}$, respectively, and individuals' end-of-period holdings by $\theta_t^{a,h} = (\theta_{1,t}^{a,h}, \theta_{2,t}^{a,h})$ for $a \in \{y, m\}$.

In a competitive equilibrium, all agents in the OLG economy choose security holdings to maximize their expected utility subject to their standard intertemporal budget constraints (depending on the market specification) and all security markets clear in each time period. For our analysis of equilibria we follow the analysis in Huffman (1987) and define agents' beginning-of-period "cash at hand." If markets are incomplete, then the middle-aged agents begin their second period in the economy with a cash-at-hand position of

$$\kappa_t^{m,h} = \phi_{t-1}^{y,h}(p_t + d_t).$$

If markets are complete, then the middle-aged agents begin their second period with cash at hand of

$$\kappa_t^{m,h} = \phi_{t-1}^{y,h}(p_t + d_t) + \theta_{s_t,t-1}^{y,h}$$

if the state s_t occurs. The expressions for the cash-at-hand positions $\kappa_t^{o,h}$ of the old agents are analogous. We define a cohort's cash at hand by $\kappa_t^a = \kappa_t^{a,1} + \kappa_t^{a,2}$ for $a \in \{m, o\}$.

The assumption of logarithmic utility implies both for the incomplete-markets and the complete-markets economy that the agents consume[1]

$$(1)\quad c_t^{y,h} = \frac{1}{3} e_t^h, \quad c_t^{m,h} = \frac{1}{2}\kappa_t^{m,h}, \quad c_t^{o,h} = \kappa_t^{o,h},$$

for $h = 1, 2$. The young agents invest the remaining $\frac{2}{3} e_t$ of their initial endowment, and the middle-aged agents invest the remaining $\frac{1}{2}\kappa_t^m$ of their cash at hand. Since the stock is in unit net supply, the stock price is the sum of all investments and, thus, satisfies the condition

$$(2)\quad \begin{aligned} p_t &= \frac{2}{3} e_t + \frac{1}{2}\kappa_t^m \\ &= \frac{4}{3} e_t + d_t - \kappa_t^o \end{aligned}$$

for both specifications of financial markets. The last two expressions show that the stock price depends on the cash-at-hand distribution in the economy. The stock price is high if the middle-aged (old) agents have a large (small) cash-at-hand position. And conversely, the stock price is low if the middle-aged (old) agents have a small (large) cash-at-hand position. The stock price, in turn, determines the share of the stock that the young agents can afford and thereby the wealth distribution in the subsequent period. The higher the stock price, the smaller the share of the stock that the young agents buy and the poorer these agents are in the subsequent period when they are middle-aged. As a result, the cash-at-hand positions of two subsequent middle-aged cohorts will be inversely related.

Conditions (1) and (2) hold for both incomplete and complete markets. The evolution

[1] In an online technical Appendix, we derive all numbered equations in this article.

of the cash-at-hand positions κ_t^m and κ_t^o over time, however, depends critically on the set of available securities. Therefore, in our analysis of these positions we now need to distinguish between the two market specifications.

A. *Incomplete Markets*

If the stock is the only security in the economy, then the stock holdings of the young satisfy $p_t(\phi_t^{y,1} + \phi_t^{y,2}) = \frac{2}{3} e_t$. This aggregate stock investment of the young agents leads to equilibrium aggregate cash-at-hand positions of the middle-aged agents evolving according to the rule

$$(3)\quad \kappa_{t+1}^m = \frac{1}{\frac{1}{2} + \frac{3}{4}\frac{\kappa_t^m}{e_t}} \left(\frac{2}{3} e_{t+1} + d_{t+1}\right).$$

This law of motion does not depend on individuals' beliefs. In the economy with incomplete markets, belief heterogeneity affects neither stock prices nor the cohorts' aggregate consumption allocations.

B. *Complete Markets*

In the economy with complete markets, the prices $q_{1,t}$ and $q_{2,t}$ of the two Arrow securities satisfy the first-order conditions of all agents trading in period t. The Euler equations of the young agents require

$$\pi_{s_{t+1}}^h c_t^{y,h} = q_{s_{t+1},t} c_{t+1}^{m,h}$$

for $h = 1,2$ and $s_{t+1} \in \{1,2\}$. Combining these equations with condition (1) and market-clearing requirements leads to the following evolution of the cash-at-hand positions of the middle-aged agents,

$$(4)\qquad \kappa_{t+1}^{m,h} = \frac{\pi_{s_{t+1}}^h e_t^h\left(\frac{4}{3} e_{t+1} + 2d_{t+1}\right)}{\pi_{s_{t+1}}^1 e_t^1 + \pi_{s_{t+1}}^2 e_t^2 + D}$$

with $D = \frac{3}{2}\left(\pi_{s_{t+1}}^1 \kappa_t^{m,1} + \pi_{s_{t+1}}^2 \kappa_t^{m,2}\right)$. Contrary to equation (3) in the incomplete-markets economy, individuals' beliefs now affect the evolution of cash-at-hand positions and, therefore, the equilibrium stock price.

II. The Effect of Financial Innovation

For an illustration of the joint impact of heterogeneous beliefs and financial innovation on asset price volatility, we impose additional assumptions on our OLG economies. We assume that beliefs are symmetric around $\frac{1}{2}$, agents of the first type in all cohorts have beliefs $(\pi_1^1, \pi_2^1) = (\eta, 1-\eta)$, and agents of the second type have beliefs $(\pi_1^2, \pi_2^2) = (1-\eta, \eta)$ for the two possible exogenous states with $\eta \in \left[\frac{1}{2}, 1\right)$. In addition, we assume that $d(s_t) = \frac{1}{3} e(s_t)$ and $e^1(s_t) = e^2(s_t) = \frac{1}{2} e(s_t)$. This last assumption implies that in each period the two young agents have identical shock-dependent endowments and differ only in their beliefs about the exogenous shocks.

The aggregate state-dependent endowment in the economy is $e_t + d_t = \frac{4}{3} e(s_t)$. Condition (1) implies that the joint consumption of all middle-aged and old agents is $e(s_t)$. For our analysis we define a normalized aggregated cash-at-hand position of the middle-aged agents by $\hat{\kappa}_t^m = \frac{\kappa_t^m}{e_t}$.

A. *Incomplete Markets*

Following condition (3), the evolution of the normalized aggregated cash-at-hand position of the middle-aged agents satisfies

$$(5)\qquad \hat{\kappa}_{t+1}^m = \frac{1}{\frac{1}{2} + \frac{3}{4}\hat{\kappa}_t^m}.$$

We observe that the normalized cash at hand $\hat{\kappa}_{t+1}^m$ does not exhibit any conditional volatility and, in fact, converges deterministically to a unique steady state $\hat{\kappa}^\star = \frac{1}{3}(\sqrt{13} - 1)$. In the long run, the stock price assumes only two different values which depend on the exogenous shock,

$$p_t = \left(\frac{2}{3} + \frac{1}{6}\left(\sqrt{13} - 1\right)\right) e(s_t)$$
$$\approx 1.10093\, e(s_t),$$

and so the stock price as well as the cash-at-hand positions of the middle-aged and old agents are linear functions of the shock-dependent endowments. As a result, the same is true for the consumption allocations of all individuals in the incomplete-markets economy. In the extreme case of no aggregate uncertainty, that is, for $e(1) = e(2)$, all endogenous variables are constant over time.

B. *Complete Markets*

Conditions (1) and the Euler equations for the young agents lead to the consumption and cash-at-hand ratios

$$\frac{c_{t+1}^{m,1}}{c_{t+1}^{m,2}} = \frac{\kappa_{t+1}^{m,1}}{\kappa_{t+1}^{m,2}} = \begin{cases} \frac{\eta}{1 - \eta} & \text{if } s_{t+1} = 1, \\ \frac{1-\eta}{\eta} & \text{if } s_{t+1} = 2. \end{cases}$$

Similarly, the Euler equations for the middle-aged agents lead to the ratios

$$\frac{c_{t+1}^{o,1}}{c_{t+1}^{o,2}} = \frac{\kappa_{t+1}^{o,1}}{\kappa_{t+1}^{o,2}} = \begin{cases} \frac{\eta^2}{(1-\eta)^2} & \text{if } s_{t+1} = s_t = 1, \\ 1 & \text{if } s_{t+1} \neq s_t, \\ \frac{(1-\eta)^2}{\eta^2} & \text{if } s_{t+1} = s_t = 2. \end{cases}$$

Contrary to the incomplete-markets economy, the beliefs of the agents now affect the agents' cash-at-hand positions and their consumption allocations. The evolution of the normalized aggregated cash-at-hand position of the middle-aged agents satisfies

$$(6) \qquad \hat{\kappa}_{t+1}^{m} = \frac{1}{\frac{1}{2} + \frac{3}{4}\Pi_{t+1}\hat{\kappa}_t^m}$$

with

$$\Pi_{t+1} = \begin{cases} 2(\eta^2 + (1-\eta)^2) & \text{if } s_{t+1} = s_t, \\ 4\eta(1-\eta) & \text{if } s_{t+1} \neq s_t. \end{cases}$$

For $\eta = \frac{1}{2}$, all expressions reduce to the corresponding terms for the incomplete-markets economy. Identical beliefs lead to identical cash-at-hand positions and consumption allocations as well as identical security prices in the two economies. For $\eta > \frac{1}{2}$ the beliefs term Π_{t+1} matters for the equilibrium, since it now depends on the sequence of exogenous shocks. As a result, over time normalized cash at hand $\hat{\kappa}_t^m$ fluctuates indefinitely. Moreover, we observe once again that the normalized cash-at-hand positions of two successive middle-aged cohorts are inversely related; the larger (smaller) $\hat{\kappa}_t^m$, the smaller (larger) $\hat{\kappa}_{t+1}^m$. This observation allows us to determine the range $(\underline{\kappa}, \overline{\kappa})$ of possible values for normalized cash at hand; the bounds must satisfy the following relationships,

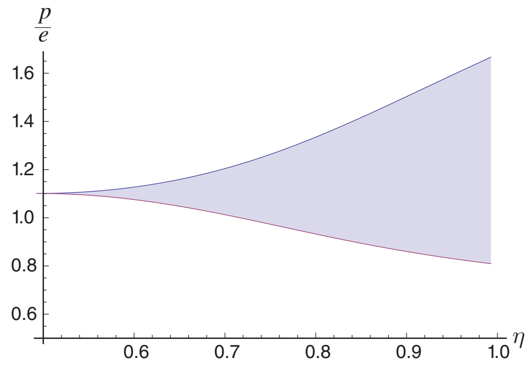

FIGURE 1. RANGE OF VALUES FOR THE NORMALIZED STOCK PRICE $\frac{p_t}{e_t}$

$$\overline{\kappa} = \frac{1}{1 + 3\eta(1-\eta)\underline{\kappa}},$$

$$\underline{\kappa} = \frac{1}{1 + \frac{3}{2}(\eta^2 + (1-\eta)^2)\overline{\kappa}}.$$

Substituting the resulting values $\underline{\kappa}$ and $\overline{\kappa}$ into the pricing equation (2) yields the range of possible values for the normalized stock price $\frac{p_t}{e_t}$. Figure 1 displays this range as a function of $\eta \in \left[\frac{1}{2}, 1\right)$.

The figure shows that as belief heterogeneity increases, stock prices start to display substantial volatility which is unrelated to aggregate shocks. To illustrate this effect, we consider a simulated time series of stock prices. We suppose that $e(1) = 1.02$, $e(2) = 0.98$, and that $\eta = 0.8$, so aggregate shocks are relatively small, and belief heterogeneity is substantial. Figure 2 shows two simulated time series over 50 periods when the economy starts in the incomplete-markets steady state.

The dashed line shows the stock price if markets remain incomplete for all 50 periods; the price is always a multiple of the aggregate endowments. The solid line, starting in period 20, displays the stock price if markets are completed in that period and remain complete thereafter. After the financial innovation,

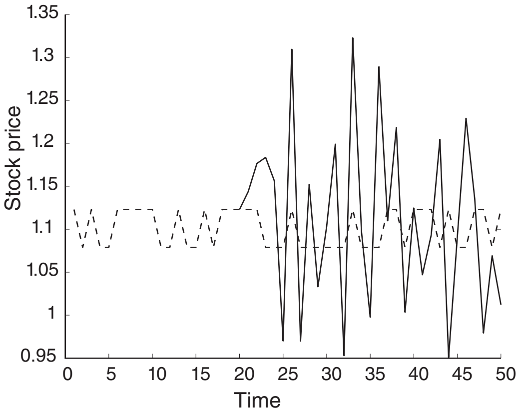

Figure 2. Simulation: Incomplete (- -) versus Complete Markets (—)

the stock price becomes much more volatile. By moving the wealth distribution, trades resulting from belief heterogeneity have a much stronger impact on the stock price than the exogenous endowment shocks.

III. Discussion

When the individuals in our economy assign different probabilities to the two possible states in the next period, then they would like to bet on (against) the state they consider to be more (less) likely. In the incomplete-markets economy, the agents cannot make such bets. They can only trade the stock to transfer wealth from the present to the future. As a result, in the long run the different individuals within a cohort have identical cash-at-hand positions and consume identical allocations. Their beliefs do not affect the equilibrium. The situation changes dramatically when markets are completed. Now the set of securities is sufficiently rich for the agents with different beliefs to speculate. In particular, an agent can transfer more wealth into a future state that is subjectively more likely and less wealth into a less probable state. Agents' beliefs now affect the equilibrium, and so the cash-at-hand positions and consumption allocations of individuals with different beliefs within the same cohort differ. Moreover, the aggregate cash at hand of middle-aged agents fluctuates over time. Since the wealth distribution in the OLG economy is important for asset prices, the wealth fluctuations lead to substantial asset price volatility.

The model in this paper is rather stylized. An individual participates for only three periods in the economy and receives an endowment only in the first period. Nevertheless, the insights from this simple OLG economy are robust. Kubler and Schmedders (2011) consider an extension of this economy in which agents live for many periods and receive labor endowment throughout their lives. Differing beliefs within cohorts leads to large asset price volatility.

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